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Low Density Spreading Multiple Access

Mohammed Al-Imari and Muhammad Ali Imran

Abstract The need for ubiquitous coverage and the increasing demand for high data rate services, keeps constant pressure on the cellular network infrastructure. There has been intense research to improve the system spectral efficiency and coverage, and a significant part of this effort focused on developing and optimizing the multiple access techniques. One such technique that been recently proposed is the Low Density Spreading (LDS), which manages the multiple access interference to offer efficient and low complexity multiuser detection. The LDS technique has shown a promising performance as a multiple access technique for cellular systems.

This chapter will give an overview on the LDS multiple access technique. The motivations for the LDS design will be highlighted by comparing it to conventional spreading techniques, including brief history of the early work on LDS. Furthermore, a background on the design of LDS in multicarrier communications, such as signatures design, a belief propagation multiuser detection, etc., will be presented along with the challenges and opportunities associated with the multicarrier LDS multiple access.

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1 Motivations for Low Density Spreading

In this section, the motivation behind the proposal of low density spreading as a multiple access technique will be presented. The aim is to provide a general understanding of the challenges/drawbacks of the conventional spreading multiple access technique that can be resolved with the LDS scheme. To this end, the CDMA system is presented with focus on the Multiple Access Interference (MAI) drawback and the approaches that have been proposed to tackle this problem. This is followed by introducing the concept of low density spreading for CDMA and highlighting its difference and advantages comparing to the conventional CDMA system.

1.1 Code Division Multiple Access (CDMA)

A spread spectrum system is a system in which the transmitted signal is spread over a wide frequency band, much wider than the bandwidth required to transmit the information being sent. The power spectral density of the useful signal will be reduced to a level even maybe below the noise level. There are many applications and advantages for spreading the spectrum; anti-jamming, interference rejection, low probability of intercept and multiple access. The primary spread spectrum system for multiple access is the Direct Sequence CDMA. CDMA is an efficient multiple access technique that has been adopted in 3G mobile communication systems [1]. CDMA allows for many sources' (or users') information to be transmitted simultaneously over a single communication channel. The sources are distinguished by spreading codes. Apart from its many advantages, CDMA also has several limitations. One of the well-known disadvantages is that CDMA is an interference limited system. This means that the capacity of CDMA system is affected by the existing of MAI.

Consider an uplink synchronous CDMA system with set of users \mathcal{K} and processing gain L . Each user will be assigned a unique spreading code with length L to spread its symbols. Let $a_k \in \mathcal{X}$ and $\mathbf{s}_k = [s_{k,1}, s_{k,2}, \dots, s_{k,L}]^T$ be the modulation symbol and the spreading code of the k th user, respectively, where \mathcal{X} is the constellation alphabet. The discrete-time model for the received signal on the l th chip, r_l , will be

$$r_l = \sum_{k \in \mathcal{K}} a_k s_{k,l} + z_l, \quad (1)$$

where z_l is Gaussian noise. In conventional CDMA systems, at the detection of a user signal, the signal of the other users are considered as interference. For the k th user the interference component can be shown as follows

$$\mathbf{r} = a_k \mathbf{s}_k + \overbrace{\sum_{m \in \mathcal{K} \setminus k} a_m \mathbf{s}_m}^{\text{mai}_k} + \mathbf{z}, \quad (2)$$

where \mathbf{mai}_k stands for the MAI the k th user sees, $\mathbf{r} = [r_1, r_2, \dots, r_L]^T$ and $\mathbf{z} = [z_1, z_2, \dots, z_L]^T$. Thus, the performance of the data detection is highly affected by the number of active users in the system. Also, with conventional detection, a power control is required to mitigate the near-far effect problem. Many approaches are proposed for alleviating the MAI and improve the system performance such as

- Employing Multiuser Detection (MUD) techniques at the receiver by exploiting the knowledge of the user of interest as well as interferers.
- Designing suitable spreading codes so that the MAI can be decreased. It is typically done by designing the codes with good cross-correlation properties.
- Incorporating Forward Error Correction (FEC) coding. Although the FEC is designed to remove the noise instead of MAI, when the MAI can be assumed as noise, a powerful FEC can be deployed to combat the MAI too.

1.1.1 Multiuser Detection

There has been great interest in improving CDMA performance through the use of MUD. Many MUD techniques are proposed to suppress the interference caused by the non-orthogonality of codes. In MUD, codes, delays, amplitudes and phases information of all the users are jointly used to better detect each individual user. The optimal MUD has been proposed by Verdu [22] and achieves the optimal performance in terms of error probability. The optimal MUD uses the Maximum Likelihood (ML) criterion in detecting the users' symbols, which selects the symbols' sequence, $\hat{\mathbf{a}}$, that maximizes the likelihood function $p(\mathbf{r}|\mathbf{a})$ given the channel observation

$$\hat{\mathbf{a}} = \arg \max_{\mathbf{a} \in \mathcal{X}^K} \left(-\|\mathbf{r} - \mathbf{S}\mathbf{a}\|^2 \right), \quad (3)$$

where $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_K]$, and $\mathbf{a} = [a_1, a_2, \dots, a_K]^T$. By incorporating the prior distribution of the transmitted symbols, the Maximum A Posteriori Probability (MAP) can be derived. The MAP detector maximizes the joint posterior probability, $p(\mathbf{a}|\mathbf{r})$, of the transmitted symbols, which can be implemented jointly

$$\hat{\mathbf{a}} = \arg \max_{\mathbf{a} \in \mathcal{X}^K} p(\mathbf{a}|\mathbf{r}), \quad (4)$$

or individually through marginalizing the joint posterior probability in (4) as follows

$$\begin{aligned} \hat{a}_k &= \arg \max_{b \in \mathcal{X}} \sum_{\mathbf{a} \in \mathcal{X}^K}^{a_k=b} p(\mathbf{a}|\mathbf{r}) \\ &= \arg \max_{b \in \mathcal{X}} \sum_{\mathbf{a} \in \mathcal{X}^K}^{a_k=b} \prod_{k \in \mathcal{K}} p(a_k) \prod_{l=1}^L p(r_l|\mathbf{a}), \quad \forall k \in \mathcal{K}, \end{aligned} \quad (5)$$

where $p(a_k)$ is the priori probability of the symbol of the k th user. The second equality based on the Bayes' rule and the fact that the users' symbols are inde-

pendent and the noise vector is independent and identically distributed (i.i.d). The optimal solution for (3) and (5) requires a search over all the possible combinations of the symbols in the vector \mathbf{a} . Hence, the computational complexity of the optimal MUD increases exponentially with the number of users, and a complexity order of $\mathcal{O}(|\mathcal{X}|^K)$ is required. Thus, it is clear that the optimal MUD is infeasible for practical implementation.

The high complexity of the optimal MUD has motivated the search for suboptimal MUD techniques with low computational complexity such as linear Minimum Mean Square Error (MMSE), the decorrelator detector, the orthogonal multiuser detector and subtractive interference cancellation detectors. In spite of the low complexity of the linear receivers, their performance can be far from the optimal MUD performance.

1.1.2 Spreading Codes Design

In CDMA, the system loading is given by the ratio of the number of admissible users K and the processing gain L (number of chips per symbol), and it is denoted by $\beta = K/L$. Thus the system is called underloaded, fully-loaded and overloaded when $\beta < 1$, $\beta = 1$ and $\beta > 1$, respectively. For underloaded and fully-loaded conditions, orthogonal codes are optimal and can be easily constructed, subsequently the multiuser channel will decouple, ideally, into single-user channels. Hence, there is no MAI and the matched filter single-user detector is optimal.

However, for an overloaded system, where K is larger than L , orthogonal codes are impossible to be constructed and non-orthogonal codes are used instead. In this case, the source of interference is due to the non-orthogonality of users' codes and the performance of the system will depend on the cross-correlation among users' codes. The codes that meet the Welch-Bound-Equality (WBE) are known as the optimal codes that maximize the sum-rate capacity for overloading condition [24]. However, the WBE-optimized codes are constructed by using a function of a specific number of active users and spreading gain. This is the problem that fundamentally limits their practicality in the real system where the number of users dynamically changes over a period of time. Furthermore, the optimality of WBE codes can be achieved only when the optimal MUD techniques are used, (e.g. MAP) [12]. Although optimal MUD techniques can effectively combat the MAI problem, its complexity grows exponentially with the number of users, and is intractable for practical implementation.

Alternatively, another scheme for designing codes for overloaded CDMA is the Hierarchy of Orthogonal Sequences (HOS). In HOS, a group of users ($K^{[1]} = L$) are assigned orthogonal codes and the rest of users ($K^{[2]} = K - L$) are assigned another set of orthogonal codes or Pseudo-random Noise (PN) sequences. Thus, the interference levels of the users are decreased significantly as compared to random spreading, since any user suffers from interference caused by the users belonging to the other group of users only. If orthogonal codes are assigned for the rest of the users, the system is referred to it as OCDMA/OCDMA, and if PN sequences are

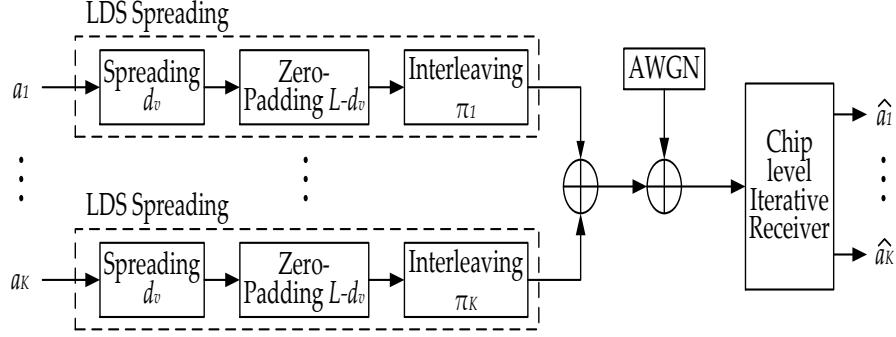


Fig. 1 Uplink LDS-CDMA block diagram.

used then it is called PN/OCDMA. It has been shown that OCDMA/OCDMA has performance close to the system that is based on WBE codes.

1.2 Low Density Spreading CDMA

Motivated by the facts mentioned in the previous section and in the pursuit to develop low complexity and efficient MUD techniques, Kabashima has proposed a low complexity MUD based on Belief Propagation (BP) using Gaussian approximation [11]. It was shown numerically that the proposed MUD achieves near optimal performance for moderately loaded CDMA system. Improvements of the algorithm have been proposed in [18]. The same approach of MUD based on the BP was proposed to approximate the parallel interference cancelation in CDMA system [21]. Inspired by the success of Low Density Parity Check (LDPC) codes, the Low Density Spreading (LDS) was proposed as a method for guaranteeing the convergence of BP based MUD [16, 26, 20, 10]. The LDS structure is also known as sparsely-spread-CDMA, sparse-CDMA and low density signature-CDMA.

The main idea of the LDS technique is to switch off a large number of spreading code chips, which makes the code a sparse vector. In other words, each user will spread its data over a small number of chips. Then the chips are arranged intelligently so that the number of interferers per chip is far smaller than the number of total users. It is proven in [16] that, with low density codes, the optimal MAP symbol detection (4) can be implemented using BP, i.e. the MUD based on BP is asymptotically optimal. The LDS-CDMA system model is depicted in Figure 1. This LDS structure brings about the following advantages:

- Higher chip-level Signal to Interference-plus-Noise Ratio (SINR) can be achieved which leads to a better detection process.
- At each received chip, a user will have relatively small number of interferers, so the search-space should be smaller and, hence, more affordable detection techniques can be used.

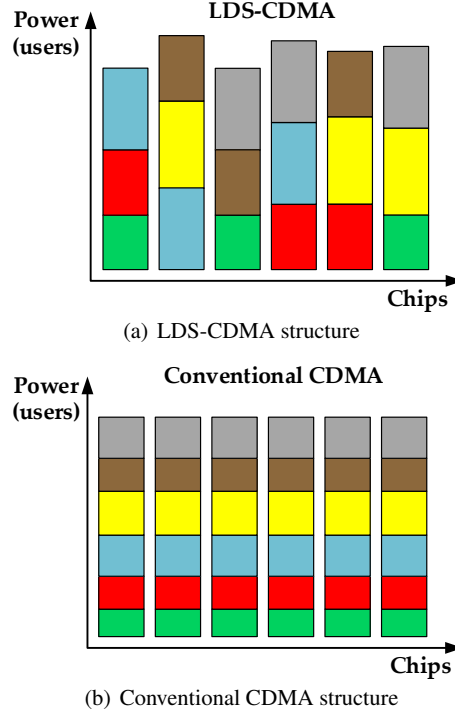


Fig. 2 Structure of LDS-CDMA in comparison to conventional CDMA.

- Each user will see an interference coming from different users at different chips which result in interference diversity by avoiding strong interferers to corrupt all the chips of a user.

Figure 2 depicts an example of the structure of the LDS-CDMA in comparison to conventional CDMA. Let d_v and d_c be the number of chips over which a single user will spread its data and the maximum number of users that are allowed to interfere within a single chip, respectively. The new spreading sequences for each user will have a maximum of d_v non-zero values and $L - d_v$ zero values. Each user will see interference coming from $d_c - 1$ users at each chip. Moreover, as shown in the figure, in LDS-CDMA the interference level is different for different chips. In conventional CDMA, each user will spread on all the L chips, so each user will see the interference coming from $K - 1$ users. LDS structure allows applying close to optimal chip-level MUD based on Message Passing Algorithm (MPA) [13]. The complexity of the LDS detector receiver turns out to be $\mathcal{O}(|\mathcal{X}|^{d_c})$, which is significantly reduced comparing to $\mathcal{O}(|\mathcal{X}|^K)$ for an optimal receiver for conventional CDMA system.

Figure 3 shows the Bit Error Rate (BER) performance for un-coded LDS-CDMA with BPSK modulation and different system loading in comparison with single-user

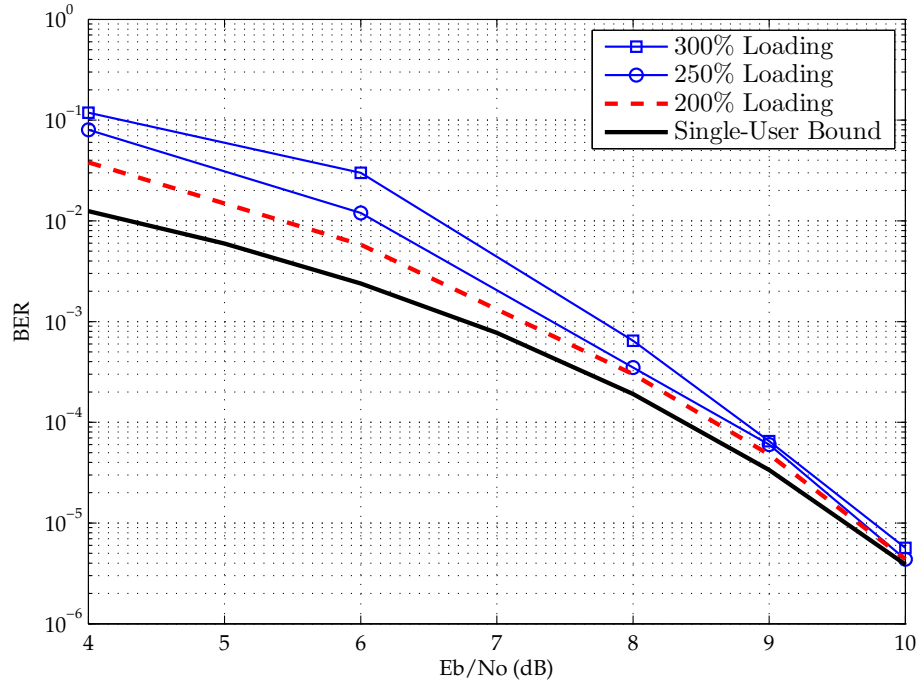


Fig. 3 Performance results for LDS-CDMA.

performance over Additive White Gaussian Noise (AWGN) channel. As the figure shows, LDS-CDMA can achieve an overall performance, in overloading conditions, that is close to single-user performance.

2 Multicarrier Low Density Spreading Multiple Access

To cope with the multipath effect of the wireless channel, a multicarrier version of LDS-CDMA has been introduced. In this section, the basic concepts of the Multicarrier Low Density Spreading Multiple Access (MC-LDSMA) system are introduced along with the system model and the receiver structure. Then, the properties of MC-LDSMA such as complexity and frequency diversity are discussed and compared with existing multiple access techniques.

2.1 MC-LDSMA System Model

Like all single-carrier communication techniques, LDS-CDMA is prone to the multipath channel conditions. The multipath propagation will result in ICI, which destroys the low density structure and a dense graph will be resulted at the receiver. Therefore, the LDS structure is applied to a multicarrier system (such as OFDM) to cope with the multipath channel effect. This can be understood as the extension of LDS structure to Multicarrier Code Division Multiple Access (MC-CDMA) system, in which the spreading is carried out in the frequency domain. As this technique uses multicarrier transmission and the users access the system through the LDS codes, it has been referred to as Multicarrier Low Density Spreading Multiple Access (MC-LDSMA) [5, 4] or LDS-OFDM [3]. The conceptual block diagram of an uplink MC-LDSMA system is depicted in Figure 4. The system consists of a set of users $\mathcal{K} = \{1, \dots, K\}$ transmitting to the same base station where the base station and each user are equipped with a single antenna. In MC-LDSMA, the user data is spread in the frequency domain before transmission. Let \mathbf{a}_k be a data vector of user k consisting of M_k modulated data symbols and denoted as

$$\mathbf{a}_k = [a_{k,1}, a_{k,2}, \dots, a_{k,M_k}]^T. \quad (6)$$

Without loss of generality, all users are assumed to take their symbols from the same constellation alphabet \mathcal{X} . Each user will be assigned a spreading matrix \mathbf{S}_k , which consists of M_k LDS codes

$$\mathbf{S}_k = [\mathbf{s}_{k,1}, \mathbf{s}_{k,2}, \dots, \mathbf{s}_{k,M_k}], \quad (7)$$

where each LDS code, $\mathbf{s}_{k,m} \in \mathcal{C}^{N \times 1}$, is a sparse vector consisting of N chips. Among these N chips only d_v chips have non-zero values, where d_v is the effective spreading factor. Each data symbol, $a_{k,m}$, will be spread using the m th spreading sequence. Let $\mathbf{x}_k = [x_{k,1}, x_{k,2}, \dots, x_{k,N}]^T$ denote the chips vector belonging to user k after the spreading process, which is given by

$$\mathbf{x}_k = \mathbf{S}_k \mathbf{a}_k. \quad (8)$$

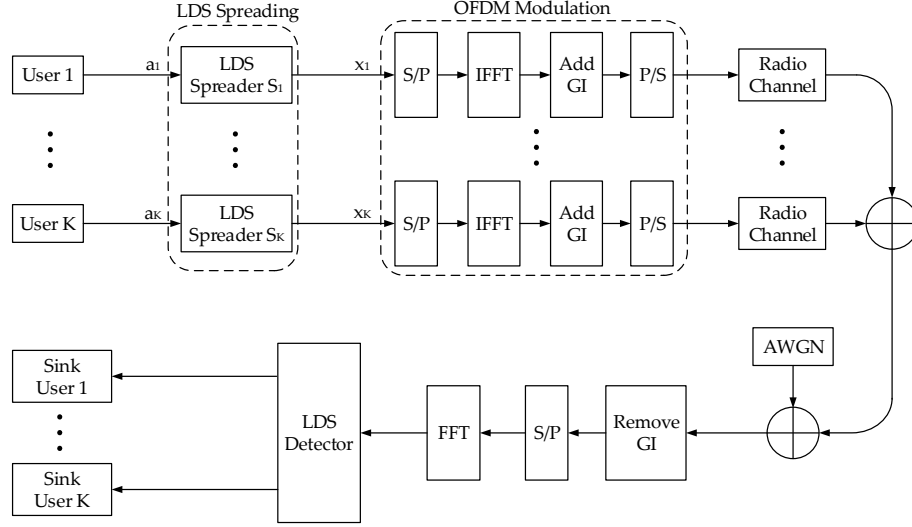


Fig. 4 Uplink MC-LDSMA block diagram.

Hence, the whole system code matrix has N rows and M columns each containing a unique spreading code, where M can be calculated as follows

$$M = \sum_{k=1}^K M_k. \quad (9)$$

The system loading (β), which is the ratio of the number of transmitted symbols to the total number of subcarriers will be

$$\beta = \frac{M}{N}$$

In orthogonal transmission, the system loading cannot be more than 1, i.e. the total number of transmitted symbols is equal or less than the number of subcarriers. The main advantage of the low density spreading is that the system loading can be more than 100% ($\beta > 1$) with affordable complexity and close to the single-user performance. The case when the system loading is larger than 1 is referred to it as an overloaded system. Each user-generated chip will be transmitted over a subcarrier of the OFDM system, and the terms chip and subcarrier will be used interchangeably to refer to the same thing.

Figure 5 illustrates the MC-LDSMA principle by an example of a system with four subcarriers ($N = 4$), serving three users ($K = 3$) with two data symbols per user ($M_1 = M_2 = M_3 = 2$), which means 150% loading. Here the effective spreading factor is two ($d_v = 2$) and each three chips sharing one subcarrier ($d_c = 3$), where d_c denotes the number of users interfere in each subcarrier. The figure shows in more details the process of low density spreading. As it can be observed, each chip

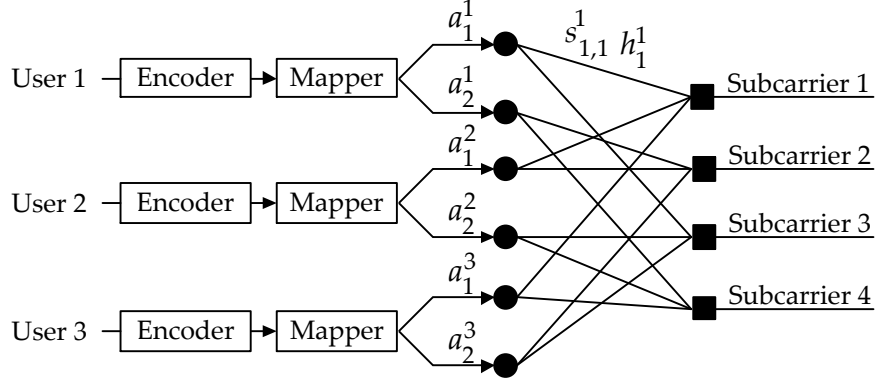


Fig. 5 Graphical representation of MC-LDSMA.

represents a subcarrier of OFDM modulation and the data symbols using the same subcarrier will interfere with each other.

The system spreading matrix can be represented by an indicator matrix $\mathbf{I}_{LDS,4 \times 6}$, which represents the positions of the non-zero chips in each spreading code as follows

$$\mathbf{I}_{LDS,4 \times 6} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

As users are not bounded to exclusively use the subcarriers, at the receiver side, users' signals that use the same subcarrier will be superimposed. However, the number of users interfere in each subcarrier is much less than the total number of users, $d_c \ll K$.

At the receiver, after performing OFDM demodulation operation, the received signal, $\mathbf{r} \in \mathbb{C}^{N \times 1}$, is given by

$$\mathbf{r} = \sum_{k \in \mathcal{K}} \mathbf{H}_k \mathbf{x}_k + \mathbf{z}, \quad (10)$$

where $\mathbf{z} = [z_1, \dots, z_N]^T$ is the Additive White Gaussian Noise (AWGN) vector and \mathbf{H}_k is the frequency domain channel gain matrix of user k

$$\mathbf{H}_k = \text{diag}(\sqrt{h_{k,1}}, \sqrt{h_{k,2}}, \dots, \sqrt{h_{k,N}}). \quad (11)$$

Here, $h_{k,n}$ is the channel gain of user k on subcarrier n . This signal \mathbf{r} is passed to the LDS MUD to separate users' symbols which is done using chip-level iterated MUD based on Message Passing Algorithm (MPA).

The basic form of chip-level iterated MUD can be explained as follows. Let

$$\mathcal{J}_n = \{(k, m) : s_{k,m}^n \neq 0\}, \quad (12)$$

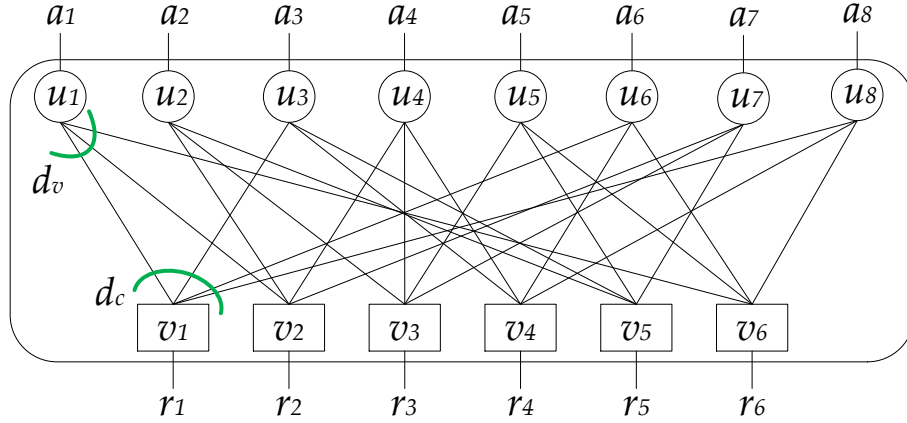


Fig. 6 Factor graph representation of the LDS structure in MC-LDSMA.

identifies different users' data symbols that share the same subcarrier n . Consequently, the received signal on subcarrier n can be written as

$$r_n = \sum_{(k,m) \in \mathcal{J}_n} a_{k,m} s_{k,m}^n \sqrt{h_{k,n}} + z_n. \quad (13)$$

A MC-LDSMA system, with M number of symbols and N number of subcarriers, can be represented using a factor graph $\mathcal{G}(\mathcal{U}, \mathcal{V})$ where users' symbols are represented by variable nodes $u \in \mathcal{U}$ and chips are represented by function nodes $v \in \mathcal{V}$. For simplicity, it will be assumed that each user transmits one symbol only. The connection between the received chip and its related users is represented by edges. Let $e_{k,n}$ represent the edge connecting variable node u_k , $k = 1, \dots, K$ and function node v_n , $n = 1, \dots, N$. It is straightforward to check that the factor graph representation of the conventional MC-CDMA is fully connected, where each variable node is connected to all function nodes. However, in MC-LDSMA system, each variable node is connected to d_v function nodes only and each function node is connected to d_c variable nodes only. Figure 6 depicts an example of factor graph representation of LDS structure in MC-LDSMA system with $K = 8$ and $N = 6$.

Using MPA, the messages are updated and iteratively exchanged between function and variable nodes along the respective edges. Those messages are the soft-values that represent the inference or the reliability of the symbol associated to each edge. Let ζ_k be the set of chip indices over which the k th symbol is spread and ξ_n be the set of symbol indices that interfering in the n th received chip, r_n . Let $L_{v_n \leftarrow u_k}$ and $L_{v_n \rightarrow u_k}$ be the message sent along edge $e_{k,n}$ from variable node u_k and function node v_n , respectively. The message $L_{v_n \leftarrow u_k}$ gives an updated inference of a_k based on the observation taken at chips $r_q, \forall q \in \zeta_k \setminus n$. The messages of the j th iteration, sent by variable nodes, are updated using the following rule

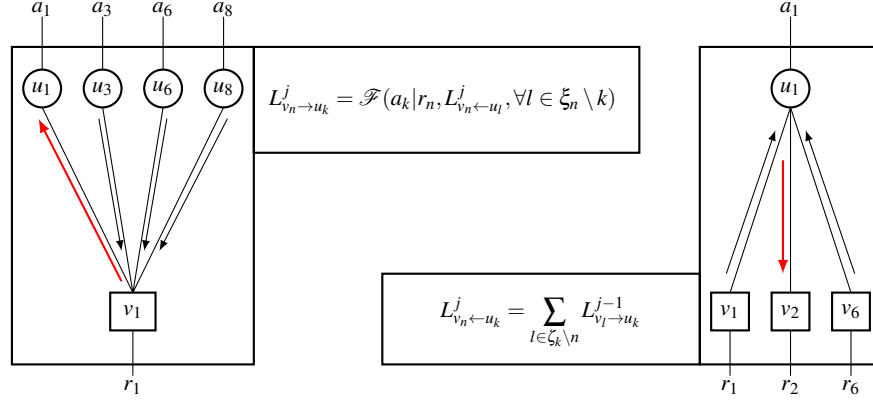


Fig. 7 Message passing process of the LDS detector in MC-LDSMA.

$$L_{v_n \leftarrow u_k}^j = \sum_{l \in \xi_k \setminus n} L_{v_l \rightarrow u_k}^{j-1}. \quad (14)$$

For n th function node the message of j th iteration is calculated as follows

$$L_{v_n \rightarrow u_k}^j = \mathcal{F}(a_k | r_n, L_{v_n \leftarrow u_l}^j, \forall l \in \xi_n \setminus k). \quad (15)$$

To approximate the optimal Maximum *A Posteriori* Probability (MAP) detector, the function $\mathcal{F}(\cdot)$ in (15) performs local marginalization and can be written as follows using the logarithmic value

$$\mathcal{F}(\cdot) = \log \left(\sum_{\mathbf{a}_n \in \mathcal{X}^{d_c}} p(r_n | \mathbf{a}_n) \prod_{l \in \xi_n \setminus k} p(a_l) \right), \quad (16)$$

where

$$p(r_n | \mathbf{a}_n) \propto \exp \left(-\frac{1}{2\sigma^2} \|r_n - \bar{\mathbf{h}}_{[n]}^T \mathbf{a}_n\|^2 \right), \quad (17)$$

$$p(a_k) = \exp(L_{v_n \leftarrow u_k}^j(a_k)). \quad (18)$$

Here, $\mathbf{a}_{[n]}$ and $\bar{\mathbf{h}}_{[n]}$ are the vectors that contain the symbols transmitted on the n th chip and their corresponding effective received code, respectively. Figure 7 illustrates the message passing process for the first function node (v_1) and the first variable node (u_1). After appropriate number of iterations or when the iterations reach the maximum limit (J), the posteriori probability of the transmitted symbol a_k will be as follows

$$L_k(a_k) = \sum_{l \in \xi_k} L_{v_l \rightarrow u_k}^J. \quad (19)$$

If hard-decision is used, the estimated value of transmitted symbol \hat{a}_k will be

$$\hat{a}_k = \arg \max_{a_k \in \mathcal{X}} L_k(a_k). \quad (20)$$

Otherwise, the soft output (19), which is calculated at each variable node, will be sent to the channel decoder.

2.1.1 LDS Codes Design for MC-LDSMA

The low density spreading codes for MC-LDSMA can be implemented by first spreading the data symbol based on the effective spreading factor (d_v), then the resulted chips are mapped to specific subcarriers. The subcarrier mapping (or it can be referred to it as subcarrier allocation) can be optimized off-line, static subcarrier allocation, or it can be optimized dynamically based on the subcarriers channel gains. In the static design of low density codes, the main aspect need to be considered is reducing the number of short-cycles in the indicator matrix. It has been shown that the MPA outputs a good approximate of the a posteriori probability if the Girth is not less than 6 [25, 8]. The Girth is the length of the smallest cycle in the factor graph associated with the indicator matrix. Thus, for better performance short-cycles of length 4, Cycle-of-Four (CoF), has to be avoided in the design of the indicator matrix. There are many algorithms in the literature for designing matrices with no CoF [14, 15], which can be used for designing LDS codes.

For LDS-CDMA these algorithms can be implemented directly. However, for MC-LDSMA, another aspect needs to be considered in the design of the indicator matrix. Due to the frequency selectivity, the subcarriers belong to each symbol has to be separated apart to achieve frequency diversity. More specifically, the subcarriers should be separated from each other by at least the channel coherence bandwidth B_c .

$$n_i - n_j \geq \left\lceil \frac{B_c}{\Delta f} \right\rceil, \quad n_i > n_j, \quad \forall n_i, n_j \in \mathcal{D} \quad (21)$$

where Δf is the subcarriers frequency separation and $\lceil x \rceil$ represent the smallest integer that is greater than x . \mathcal{D} is the set of subcarriers that the symbol will be spread on, and n is the subcarrier index.

2.2 MC-LDSMA Properties in Comparison with Other Multiple Access Techniques

In this section, the properties of MC-LDSMA are discussed and the scheme is compared with other multiple access techniques such as OFDMA and MC-CDMA. The section will focus on the link-level aspects of MC-LDSMA. The system-level aspects of MC-LDSMA and comparison with other multiple access techniques will be deferred to the chapter where radio resource allocation is considered. MC-LDSMA can represent a midway approach comparing to orthogonal multiple access tech-

niques and conventional spread-spectrum multiple access. In the orthogonal multiple access techniques, such as OFDMA and Single Carrier (SC)-FDMA, the users are orthogonal to each other, which simplifies the detection process but no frequency gain can be achieved. On the other hand, spread-spectrum multiple access methods, such as MC-CDMA, attempt to achieve the full frequency diversity gain on the cost of higher MUD complexity. In MC-LDSMA, part of the frequency diversity can be achieved by spreading on a small number of subcarriers with affordable MUD complexity. Here, the advantages of MC-LDSMA in comparison to these two conventional approaches will be highlighted.

A. Spread-Spectrum Multiple Access

In conventional MC-CDMA, each user's symbols are spread over all the subcarriers. In the synchronous MC-CDMA downlink, orthogonal spreading codes are of advantage, since they reduce the multiple access interference in comparison to non-orthogonal sequences. By contrast, in the uplink, the MC-CDMA signals received at the base station suffer from different degradations introduced by the users' independent frequency-selective channels. Consequently, users' codes are no longer orthogonal, which causes multiuser interference. Although optimal MUD techniques can effectively combat the multiuser interference, their complexity increases exponentially with the number of users, which is intractable for practical implementation. Considering that LDS structure reduces the number of interferers in each chip, it allows applying close to optimal MUD based on MPA. For MC-LDSMA the complexity of MAP receiver will turn out to be $\mathcal{O}(|\mathcal{X}|^{d_c})$, which is significantly reduced compared to $\mathcal{O}(|\mathcal{X}|^K)$ for optimal MUD for MC-CDMA. As MC-CDMA system the symbol is spread over all the subcarriers, full frequency diversity can be achieved if the optimal MUD is applied. However, due to the high complexity of the optimal MUD in MC-CDMA, linear MUD is usually implemented in practical systems. Therefore, MC-CDMA can't enjoy full diversity gain and MC-LDSMA outperforms the MC-CDMA performance as it will be shown in the next sections.

Furthermore, it is well known that, from frequency diversity perspective, the user does not need to transmit over all the subcarriers to achieve the maximum diversity. In fact, the frequency diversity can be achieved by transmitting on subcarriers which are separated from each other by the coherence bandwidth of the channel [7]. Thus, it is possible to reduce the MAI and the MUD complexity without sacrificing the diversity by spreading on less number of subcarriers comparing to the conventional MC-CDMA.

Comparing to Sparse Code Multiple Access (SCMA), MC-LDSMA and SCMA are based on the same concept of low density spreading. SCMA, which can be considered as an extension of MC-LDSMA, uses modulation shaping instead of the regular rectangular QAM modulation that is adopted in MC-LDSMA. Using modulation shaping may improve the performance of the LDS multiple access in AWGN. However, it is not clear if there is a gain in applying modulation shaping to LDS

multiple access to frequency selective channel, which is a more practical scenario in real systems compared to AWGN.

B. Orthogonal Multiple Access (OFDMA)

One example of orthogonal multiple access is OFDMA. In OFDMA system, the set of subcarriers is divided into several mutually exclusive subsets and then each subset is allocated to transmission of a user signal. This approach creates frequency domain orthogonality for users' signals when the transmitter and the receiver are perfectly synchronized and hence avoids MAI. As in OFDMA user-data symbols are assigned directly to subcarriers, the frequency domain diversity will not be achievable at the modulation symbol level. Thus, it will be crucial to incorporate properly designed error correction coding and interleaving schemes to obtain this diversity at a later stage.

On the other hand, in MC-LDSMA system, each modulation symbol is spread on a number of subcarriers. Therefore, frequency diversity is achievable at the modulation symbol level in addition to the frequency diversity gained when channel coding is used. Frequency diversity can be gained by assigning distributed and spaced-enough subcarriers for spreading of a given data symbol. Consequently, even though the spreading is over a limited number of chips, the system will still be able to gain frequency diversity. However, MC-LDSMA detector has larger complexity comparing to OFDMA receiver due to the need to implement MUD. The increased computational complexity of the system in comparison to a conventional receiver used for OFDMA is practically affordable as the added complexity is in the base station side. Also, the complexity is reasonably justified considering the achieved gain in performance. Further reduction in complexity of multiuser detection for MC-LDSMA can be achieved by applying the Grouped-based technique [23]. The technique works by arranging the interfering users of a chip into two groups and approximating the interference coming from the other group as a single symmetric Gaussian distributed variable. For LDS system with 200% loading, the loss of approximately 0.3 dB compared to its brute-force counterpart can be achieved while reducing the complexity to more than half.

3 Challenges and optimization opportunities for LDS

In this section, the challenges and opportunities associated with MC-LDSMA system that can further improve the system performance will be presented.

3.1 *Envelope Fluctuations in LDS Multiple Access*

The transmitted signals in multicarrier communication systems consist of sums of trigonometric series. The major drawback of these systems is their large envelope fluctuations. It is well known that power amplifiers are peak power limited and when the input exceeds a limit the amplifier input-output characteristics are not linear any more. Therefore, when a signal with large envelope fluctuations that exceeds the dynamic range of the amplifier is passed through the amplifier it will suffer from spectrum re-growth and in-band distortion. This will cause BER degradation and out-of-band radiation. If the amplifier is designed with high de-rating to accommodate these peaks, it tends to be very inefficient. In the downlink, the base station can tolerate higher power consumption and signal processing complexity. However, in the uplink, the envelope fluctuations can be problematic where the terminals are battery powered and cannot offer high signal processing complexity. Many techniques have been proposed to reduce the envelope fluctuations in order to mitigate the associated amplifier linearity requirements, the out-of-band radiations or the required amplifier de-rating which is necessary for preventing the amplifier saturation at high input signal peaks.

As a multicarrier technique, it is expected that MC-LDSMA inherits the large envelope fluctuations drawback. As it has been explained before, in MC-LDSMA the data symbol is spread over a small number of subcarriers. This implies that the number of used subcarriers is larger than the non-spreading case (i.e. OFDMA). Hence, it is expected that MC-LDSMA has higher PAPR/CM comparing to OFDMA. Thus, it is essential to investigate the envelope fluctuations of MC-LDSMA with conventional LDS codes. In the conventional LDS codes, random phases and random sub-carrier allocation are used.

It is shown that MC-LDSMA with conventional LDS codes has considerably high envelope fluctuations [2]. Consequently, envelope fluctuations reduction techniques are required. At first, it may be thought that the large envelope fluctuations are caused by using more subcarriers in MC-LDSMA. However, the high envelope fluctuations for MC-LDSMA is caused by assigning random phases to the LDS codes without taking into account the correlation between the chips. In MC-LDSMA, the chips that belong to the same symbol are correlated. The phases can be tuned to reduce the envelope fluctuations. In fact, there is more flexibility in tuning the chips' phases in MC-LDSMA comparing to conventional spreading techniques such as MC-CDMA. The autocorrelation and cross-correlation properties of the LDS codes do not affect the detection process. Hence, this flexibility can be

utilized and the phases can be tuned to reduce the envelope fluctuations of the MC-LDSMA signal.

However, the optimal phases that minimize the PAPR are hard to be found using classical optimization techniques [6]. Therefore, suboptimal phases to reduce the PAPR/CM of the signal have to be used. Many closely related phases have been found by different researchers that have low envelope fluctuations, such as Newman [19, 9] and Narahashi [17] phases. These phasing schemes didn't attract significant attention within the context of PAPR reduction for MC-CDMA due to the constraints of autocorrelation and cross-correlation on the codes. Furthermore, in MC-CDMA, if the user needs to send more than one symbol, it has to be multiplexed in the code domain. Consequently, the resulted signal will have high PAPR/CM despite the single code design. On the other hand, in MC-LDSMA, there are no such constraints and the symbols can be multiplexed in the frequency domain. Hence, these phases can be employed in MC-LDSMA to reduce the envelope fluctuations. The Newman phases are given by

$$\theta_n^{Newman} = \frac{\pi(n-1)^2}{N}, \quad n = 1, 2, \dots, N. \quad (22)$$

The phases of the LDS codes are adjusted according to (22) as follows

$$s_n = \frac{1}{\sqrt{|\xi|}} e^{j\theta_n}. \quad (23)$$

These phasing schemes are designed in a way requires the subcarriers allocated to the user to be equally spaced. However, restricting all the users to have equally spaced subcarrier allocation in MC-LDSMA will result in a fully connected graph, which reduce the receiver detection efficiency, and the codes cannot be classified as low density any more. Accordingly, these phases are applied to for MC-LDSMA with resource block based allocation, which is a structure that is already adopted in 3GPP-LTE system. This is done by dividing the total numbers of subcarriers into segments consist of N_z adjacent subcarriers. As the frequency segment can represent the frequency dimension of a resource block, it is referred to it as a Resource Block (RB). Therefore, instead of individual subcarriers, resource blocks are allocated to users. It is a common practice to group a number of adjacent subcarriers in both time and frequency domains in a form of a resource block. However, the time dimension of the resource block does not affect the PAPR. Figure 8 depicts the two types of subcarrier allocation schemes, resource block based and individual subcarriers based. It shows an example of 4 users with two symbols ($M = 2$) per user and effective spreading factor equal to two ($d_v = 2$). In resource block based allocation each user is allocated two resource blocks with size two ($N_z = 2$), where N_z is the number of subcarriers per resource block.

Figure 9 shows the results of Cubic Metric (CM) for MC-LDSMA when the subcarrier allocation is resource block based and compared with OFDMA and SC-FDMA. The figure shows the CM of MC-LDSMA with Newman, Narahashi and random phases. As the CM of OFDMA and MC-LDSMA is the same for different

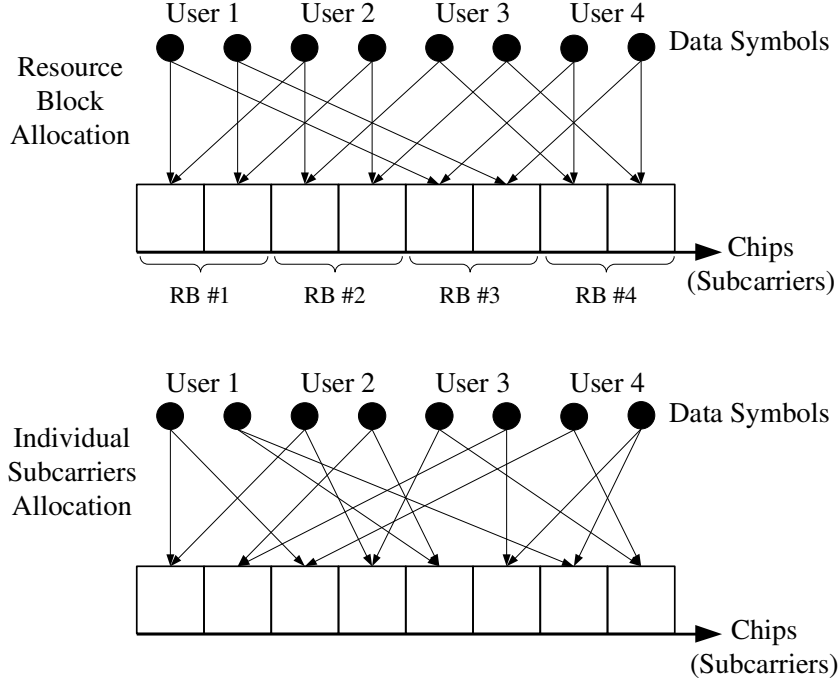


Fig. 8 Subcarrier allocation schemes for MC-LDSMA.

modulation orders, only the results of 16QAM modulation is presented in the figure. As the figure shows, Newman and Narahashi phasing schemes produce identical CM values. Both Newman and Narahashi phasing schemes significantly reduce the CM of MC-LDSMA in comparison to random phases. Random phases still suffer high CM regardless of the allocation scheme used. With phasing schemes, the CM of MC-LDSMA is reduced by 5.5 dB comparing to random phases. Comparing to SC-FDMA, MC-LDSMA has higher PAPR/CM values, especially comparing to QPSK modulation.

A major advantage of applying the phasing schemes is that it does not require modification in the MC-LDSMA system structure, and no complexity will be added to the system. In fact, generating Newman and Narahashi phases using the closed forms is less complex than the generation of random phases.

3.2 Radio Resource Allocation for Low Density Spreading

The varying nature of the wireless channel in time domain and/or frequency domain represents a challenge for reliable communication. To cope with the varying nature of the channel, dynamic radio resource allocation is usually implemented

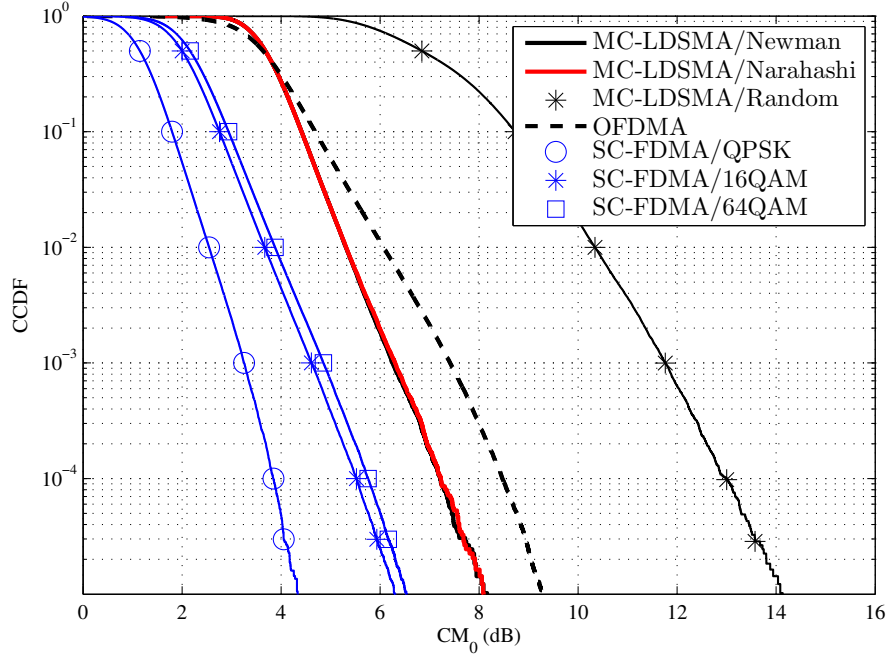


Fig. 9 CM comparison for MC-LDSMA with different phasing schemes: $N_z = 12$.

in wireless communications. The main concept of radio resource allocation is to utilize the knowledge of channel information at the transmitter to optimize the system performance. According to the channel conditions, the transmission parameters, such as transmitted power, allocated spectrum, modulation and coding orders, are adjusted to improve the system performance such as data rates, fairness, delays, etc. The low density spreading in MC-LDSMA makes the radio resource allocation more challenging compared to conventional multiple access techniques. The non-orthogonality in MC-LDSMA will couple the power allocation problem among the users, where each user's power allocation will affect other users due to the interference. Furthermore, the spreading will couple the subcarrier allocation problem among the subcarriers. Thus, the conventional radio resource allocation algorithms, optimal and suboptimal, cannot be directly applied to MC-LDSMA technique, and new analysis and algorithms need to be developed.

The single-user power allocation for LDS technique is more challenging comparing to the non-spreading case, which has a well-known water-filling solution. Assuming that N subcarriers indexed by the set $\mathcal{N} = \{1, 2, \dots, N\}$ are allocated to a user, the user will partition this set into M subsets/groups indexed by $\mathcal{M} = \{1, 2, \dots, M\}$. Each subset of subcarriers will be used to spread one symbol, and the number of symbols will be $M = N/d_v$. The single-user power allocation problem for LDS can be split into two parts: Firstly, what is the optimal subcarriers partitioning

that maximizes the user rate? Secondly, for a given subcarriers partitioning, what is the optimal power allocation?

It has been shown in [5] that for a given subcarriers partitioning scheme, the optimal power allocation algorithm can be conducted in two steps: the first step is water-filling to find the power for each symbol; the second step is the maximum ratio transmission of the symbol power over the symbol's subcarriers.

In MC-LDSMA technique, the available N subcarriers should be partitioned into M subsets, where each subset will be used to transmit one symbol. In conventional LDS codes, the subcarriers are partitioned randomly, which has shown satisfactory BER performance [4]. However, the random partitioning scheme has not been investigated from the user rate optimization perspective. The problem of partitioning the subcarriers to maximize the user rate can be formulated as follows

$$\max_{\mathcal{D}_m \in \mathcal{N}} \sum_{m \in \mathcal{M}} R_m(\mathcal{D}_m), \quad (24)$$

subject to

$$\mathcal{D}_m \cap \mathcal{D}_j = \emptyset, \quad \forall m \neq j, m, j \in \mathcal{M}. \quad (25)$$

$$|\mathcal{D}_m| = d_v, \quad \forall m \in \mathcal{M}. \quad (26)$$

This is a combinatorial optimization problem with a large search space. The number of possible LDS codes to be generated from the N subcarriers for a specific spreading factor d_v is given by

$$C_{LDS} = \frac{1}{2} \binom{2d_v}{d_v} \prod_{i=0}^{M-3} \binom{N-1-id_v}{d_v-1}. \quad (27)$$

In fact, the number possible LDS codes (C_{LDS}) represents the cardinality of the feasible search space of problem (24 - 26). In order to see how large is the search space, let us consider $N = 32$ and $d_v = 2$, so the number possible LDS codes will be $C_{LDS} = 1.92 \times 10^{17}$. It can be seen that brute-force search is unfeasible for practical systems, and partitioning schemes with low complexity need to be considered. Even though the subcarrier partitioning is a non-convex problem, there is an underlying Schur-convex structure, which can be utilized to solve the problem.

The optimal subcarrier partitioning that maximizes the user's rate with optimal power allocation is the one that gives the Most Majorized gain Vector (MMV), and can be defined as follows

$$\mathcal{D}_m^* = \{g_{\pi(m)}, g_{\pi(m+1)}, \dots, g_{\pi(m+d_v-1)}\}, \quad \forall m \in \mathcal{M}, \quad (28)$$

where π represents a permutation of the subcarrier gains in a descending order such that $g_{\pi(1)} \geq g_{\pi(2)} \geq \dots \geq g_{\pi(N)}$. In this scheme, the subcarriers are sorted in a descending order then the first best d_v subcarriers are combined to create one symbol, the second best d_v for another symbol and so on. The optimality proof of this scheme can be done by showing that this partitioning scheme will give symbols' gain-vector that majorizes any other partitioning scheme [5]. Figure 10 shows the user spectral

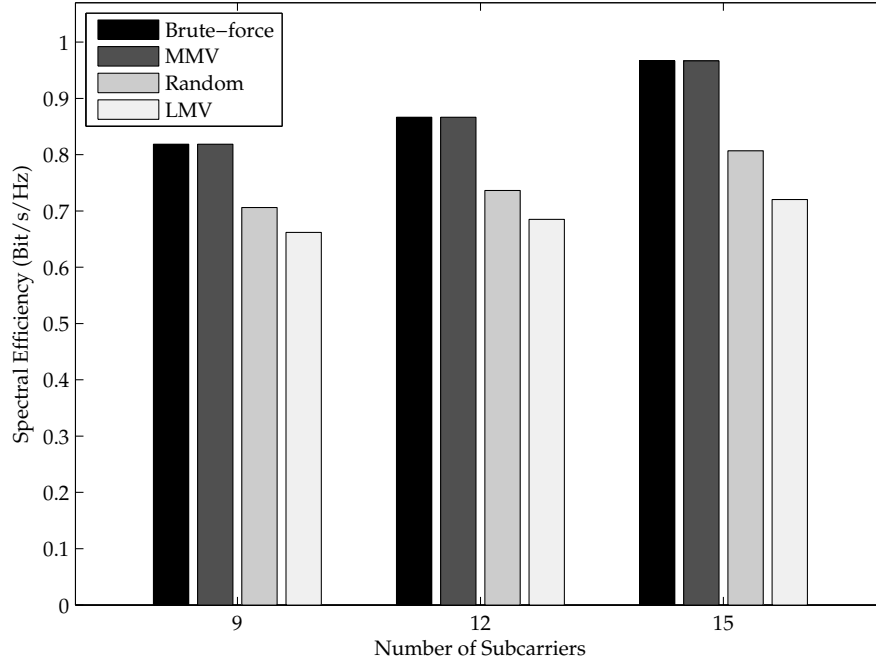


Fig. 10 Spectral efficiency comparison for different subcarriers partitioning schemes.

efficiency for different subcarriers partitioning schemes. As it can be observed from the figure, the partitioning scheme that results in the most majorized gain vector (MMV) achieves the same spectral efficiency for the brute-force search and it significantly outperforms the random and Least Majorized Vector (LMV) schemes. The random and LMV schemes only achieve 85% and 72% of the optimal solution, respectively, which indicates the sub-optimality of these schemes. Furthermore, the very poor performance of the LMV partitioning, demonstrates the importance of vector majorization in generating the symbols gains, where it shows that a less majorized gain vector will produce very low user rate.

These results show the importance of radio resource allocation in enhancing the performance of the LDS multiple access. More analysis and radio resource optimization that consider the multiuser case can be found in [5].

4 Summary

In this chapter, we provided an overview of the low density spreading multiple access technique, which is considered as a promising access scheme for future wireless communication systems. The overview of the challenges in the conventional spreading techniques, due to the multiple access interference, has provided insights on the motivations for the LDS design. Low density spreading can manage the multiple access interference and offer an efficient, low complexity multiuser detection. To cope with the multipath effect of the wireless channel, low density spreading is applied to multicarrier systems, which is referred to as Multicarrier Low Density Spreading Multiple Access (MC-LDSMA) or LDS-OFDM. Compared to orthogonal access schemes (such as OFDMA), MC-LDSMA can support higher number of users in the system. Also, due the spreading nature of LDS, better frequency diversity can be achieved which results in improved link-level performance.

In addition, the chapter discussed the challenges and opportunities associated with MC-LDSMA system such as envelope fluctuations and radio resource allocation. The impact of subcarriers' allocation schemes and the phases of the LDS codes on the PAPR/CM of MC-LDSMA signals were investigated. We have shown that with proper phasing schemes applied for the LDS codes, significant PAPR/CM reduction can be achieved. Furthermore, we presented radio resource allocation algorithms that can improve the system spectral efficiency. Numerical and simulation evaluation results have been provided to demonstrate the achieved gains by using MC-LDSMA with the optimized radio resource allocation algorithms.

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